# BIRZEIT UNIVERSITY

## MATHEMATICS DEPARTMENT

**First Exam** **MATH 2351** **Fall 2018**

**Name( In Arabic)** ...[illegible]....**Number**.....[illegible]

**Time : 75 minutes**

**Please circle your *discussion* class section**

| Section | Instructor | Day | Time |
|---|---|---|---|
| 1 | Wala'a Yasin | S | 13:00 - 13:50 |
| 2 | Duha Sharhah | W | 11:00 - 11:50 |
| 3 | Wala'a Yasin | S | 12:00 - 12:50 |
| 4 | Wala'a Yasin | W | 10:00 - 10:50 |
| 5 | Batol Rdad | S | 13:00 - 13:50 |
| 6 | Batol Rdad | S | 11:00 - 11:50 |

$$\frac{39.5}{40}$$

Excellent

7. $\lim\limits_{x\to -4}\dfrac{x^2+2x-8}{x+4}=$

a. 6
b. $-6$.
c. $\infty$.
d. $\dfrac{1}{6}$.

$\frac{0}{0}$

$\dfrac{(x+4)(x-2)}{x+4}$

$-4-2$

$-6$

8. $\lim\limits_{x\to 2^-}\dfrac{1}{x-2}=$

a. 0.
b. $\infty$.
c. $\dfrac{1}{4}$.
d. $-\infty$

$\lim\limits_{x\to 2^-}\dfrac{1}{x-2}$

$-\infty$

9. $\lim\limits_{x\to\infty}\dfrac{5x^2-8}{4-2x^2}=$

a. 0
b. $-\dfrac{5}{2}$.
c. $\infty$.
d. $-\dfrac{2}{5}$.

$\dfrac{+5}{-2}$

10. Use the marginal analysis to estimate the profit from the **101**st unit sold if $P(x)=-0.04x^2+27x-1000$

a. 19
b. 18.96
c. 18.92
d. None of the above

$P'(100)$

$P' = -2(.04)x + 27$

$P' = -.08x + 27$

11. The function $f(x)=\begin{cases} x^2-3 & \text{for } x<-2 \\ 2x+5 & \text{for } -2\le x<1 \\ 5x & \text{for } x\ge 1 \end{cases}$

a. Is continuous for all values of x.
b. Is continuous for all values of x except $x=-2$.
c. Is continuous for all values of x except $x=1$. الجواب
d. Is continuous for all values of x except $x=-2$ and $x=1$.

$x=-2$

$\lim\limits_{x\to -2^+} f(x)$

12. If the supply function is $p=x+20$, and the demand function is $p=50-x^2$, find the **equilibrium point**.

a. (5, 5)
b. (11, 3)
c. (2, 11)
d. (5, 25). الجواب
e. None of the above.

$x+20=50-x^2$

$x^2+x-30$

$x^2+x-30=0$

$(x+6)(x-5)$

$x=-6$ $x=5$

$P=5+20$

13. Let f and g be two functions satisfy $f'(4) = -2$ and $g'(4) = -3$. Find $h'(4)$ for $h(x) = 3f(x) - 2g(x)$

a. 12
b. 5
c. 6
d. 0

$h' = 3f'(x) - 2g'(x)$
$h'(4) = 3f'(4) - 2g'(4)$
$3(-2) - 2(-3)$
$-6 - -6$
$-6 + 6$
$0$

14. If $f(x) = 3(2x+1)^{\frac{3}{2}}$, find $f'(4)$.

a. $\frac{9}{2}$
b. $\frac{27}{2}$
c. 9
d. 27

$3\left(\frac{3}{2}\right)(2x+1)^{\frac{1}{2}} 2$
$9(2x+1)^{\frac{1}{2}}$
27

$3\left(\frac{3}{2}\right)(2x+1)^{\frac{1}{2}} 2$
$9(9)^{\frac{1}{2}}$
$9 \times 3$
27

$\frac{3}{2} - \frac{2}{2}$

15. A producer can sell 2000 items at a price of $10. For each **decrease** in price of $2, the producer can sell an **additional** 100 items. Write the demand equation.

a. $p = -0.02x + 14$
b. $p = -0.02x + 50$
c. $p = -0.02x + 40$
d. None of the above.

$(2000, 10)$ $(2100, 8)$
$\frac{8-10}{2100-2000} = \frac{-2}{100} = -0.02$

16. For what value(s) of x does the graph of the function $f(x) = x^3 - 3x^2 - 45x + 4$ have a **horizontal tangent**?

a. x = 3, -5
b. x = -3, 5
c. x = 0, 3
d. x = -3, -5

ميل افقي
معدل صفر

$f' = \frac{3x^2 - 6x - 45}{3}$

$y - 10 = -.02(x - 2000)$
$y - 10 = -.02x + 40$
$+10 \qquad +10$
$y = -.02x + 50$

$x^2 - 2x - 15 = 0$
$(x-5)(x+3)$
$x = 5 \quad x = -3$

23.5

**Part 2 (24 points): Show all your work**

1. The demand q for a product at a price p is $q = \frac{54}{\sqrt{2p+1}} - 1$. Find and **interpret** the **rate of change** of demand with respect to price at p = $4.

$\left.\frac{dq}{dp}\right|_{p=4}$

$q = 54(2p+1)^{-\frac{1}{2}} - 1$
$\frac{dq}{dp} = 54\left(-\frac{1}{2}\right)(2p+1)^{-\frac{1}{2}-1}(2)$
$= -27(2p+1)^{-\frac{3}{2}}(2)$
$\left.\frac{dq}{dp}\right|_{p=4} = -54(9)^{-\frac{3}{2}} = \frac{-54}{(9)^{\frac{3}{2}}} = \frac{-54}{27} = -2$

$-\frac{1}{2} - \frac{2}{2}$
$-\frac{3}{2}$

3

as the price increase by 1$ (p = 4$) the Quantity demand is decrease by 2 units

2. The revenue function for a product is given by $R(x) = 40x - 0.02x^2$. What is the **average rate of change** in revenue as x increases from 20 to 30 units.

$R(x) = \frac{R(30) - R(20)}{30 - 20}$ (20, 30)

$= \frac{40(30) - .02(30)^2 - (40(20) - .02(20)^2)}{10}$

3



$\frac{1200 - 18 - (800 - 8)}{10}$
$\frac{1200 - 18 - 800 + 8}{10} = 39$

$b = 360$ $\quad$ $V.C = 10x + 0.2x^2$

Suppose a company has **fixed cost** of $360 and **variable cost** of 10 + 0.2x dollars per unit, where x is the total number of units produced. Suppose further the **selling price** of the product is given by p = 50 – 0.2x dollars per unit.

3. Write the **cost function**.

$C(x) = V.C + f.C$

$= (10 + 0.2x)x + 360 \Rightarrow C(x) = 10x + 0.2x^2 + 360$

3

4. Write the **revenue function**.

$R(x) = px$

$R(x) = (50 - 0.2x)x \Rightarrow R(x) = 50x - 0.2x^2$

2

5. Write the **profit function**.

$P(x) = R(x) - C(x)$

$P(x) = 50x - 0.2x^2 - (10x + 0.2x^2 + 360)$

$50x - 0.2x^2 - 10x - 0.2x^2 - 360$

$P(x) = 40x - 0.4x^2 - 360$

2

6. Find the **maximum profit**.

$P(x) = 40x - 0.4x^2 - 360$

Max profit $\Rightarrow$ Vertix $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$\frac{-b}{2a} = \frac{-40}{2(-0.4)} = \frac{+40}{+0.8} = 50$

max profit is $P(50)$

$P(50) = 40(50) - 0.4(50)^2 - 360$

$= 2000 - 1000 - 360$

$1000 - 360$

Max profit $\boxed{640\$}$

3

7. What **price** maximizes the profit?

$\frac{-b}{2a} = \frac{-40}{-0.8} = \boxed{50 \text{ unit}}$

$P = 50 - 0.2(50)$

$P = 50 - 0.2(50) \Rightarrow 50 - 10$

$\boxed{= 40\$}$

3

8. Find and **interpret** $P'(20)$.

$P(x) = 40x - 0.4x^2 - 360$

$P' = 40 - 0.8x$

$P'(20) = 40 - 0.8(20) \Rightarrow 40 - 16 = \boxed{24}$

$\Rightarrow$ producing and selling 20 unit change is estimated (approximated) by 24\$

3

9. How many units should be produced to guarantee **a profit** of money?

No loss Break Even.

$P(x) = 0$

$C(x) = R(x)$

~~P(x)=0~~

$P(x) = 0 \Rightarrow 40x - 0.4x^2 - 360 = 0$

$\div 0.4$

$-100x + x^2 + 900$

$x^2 - 100x + 900 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{100 \pm \sqrt{(100)^2 - 4(900)}}{2}$

$= \frac{100 \pm 80}{2} = \boxed{90}$ $\quad$ $\boxed{10}$

$$x^2 - 100x + 900 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-100) \pm \sqrt{(100)^2 - 4(1)(900)}}{2}$$

$$= \frac{100 \pm 80}{2}$$

$$\frac{100 + 80}{2} = 90$$

$$\frac{100 - 80}{2} = 10$$

$$(x - 90)(x - 10)$$

$$x = 90 \qquad x = 10$$

2.5

Profit $\Longrightarrow$ $\boxed{10 \leq x \leq 90}$